

Mara Nord project

Work Package 5

A supplement of the final report,

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Wavelet analysis in evaluating the homogeneity of new asphalt pavement

The final report of work package 5 of the Mara Nord project describes the results of evaluating the current void content method that is used in Finland in quality assurance of new asphalt pavement. The current method is based on the use of Ground Penetrating Radar (GPR) in determining the void content of asphalt pavement together with reference drill cores that must be taken from pavement. In this report an alternative method is suggested for quality assurance which is also based on the use of GPR, but which can be performed in purely nondestructive way without reference drill cores. The new method is raised from suggestions that the quality of new asphalt pavement is related to the uniformity of compaction on a road section. As for the uniformity, it relates to homogeneous mixture of asphalt including also the void content in it. The homogeneity can be determined directly from the GPR data. Wavelet analysis gives appropriate tools for detecting significance variation from GPR data. This document is a supplement to the final report and gives additional information for the wavelet analysis technique.

Background of Wavelet Transform

The Wavelet transform is a mathematical technique to analyze nonstationary time series that contain oscillation at many different frequencies. With GPR survey the oscillation of measured pulse amplitudes or calculated dielectric values represents the time series. The Wavelet technique transforms one-dimensional oscillation in time (distance) space to the information of detected periods (frequencies) of oscillation including also their locations in the series.

There is an inverse wavelet transform which allows the original time series to be recovered from its wavelet transform. Before the inversion the transformed coefficients can be manipulated to get the original signal in a modified form where, for example, noise is removed.

Basically the wavelet transform is performed by a convolution of the original signal with a set of functions which are generated by translations and dilations of a selected wavelet function. The convolution is a mathematical operation which gives some measure for the coherence of the forms of signal and wavelet function. Translation and dilation of wavelet function must be constructed in an adequate way, and also the wavelet function needs to complete special conditions. To perform the convolution two different methods discrete (DWT) and continuous wavelet transform (CWT) can be used. The wavelet function can form an orthogonal basis, which implies that DWT need to be used. CWT can be used for both orthogonal and non-orthogonal wavelet functions. Mostly both of these methods can be used for same purposes. If the orthogonality is important for analysis, the DWT should be used. In the examples of this documents CWT with non-orthogonal Paul wavelet function is used. [1, 3]

Normalized Wavelet Power Spectrum

In Wavelet Power Spectrum (WPS) the Wavelet transform coefficient at each scale and time indexes are squared. WPS represents the power of the detected period of oscillation at a particular location. It is usually visualized as a surface plot, the Wavelet *scalogram*, where the power for a particular period and location is illustrated, for example, by different colors (see Figures 1-3, b). However, WPS is only a qualitative representation of the powers of the oscillations in the data. Torrence and Compo [1] introduce an analysis technique of WPS that may be useful for developing the quantitative measure of the homogeneity of new asphalt pavement. With this technique it is possible to normalize WPS such that different WP Spectra can be compared to each other. By [1, pp 65-68], the expectation value for coefficients in WPS of a white-noise process $|W_n(s)|^2 = \sigma^2$ at all time index n and scaling index s , where σ^2 is the variance. Therefore, the normalization of WP Spectra by $\frac{1}{\sigma^2}$ gives a measure of the power relative to white noise. White-noise time series is a random signal

that has equal power across the frequencies within a fixed bandwidth. White noise spectrum as well the other theoretical background noise spectrums can be realized computationally. If an appropriate background spectrum, for instance white noise, can be chosen, it is possible to determine the significance levels for wavelet spectra. The significance level for background noise spectrum can be used as comparative level for good homogeneity of asphalt pavement. The global wavelet spectrum (GWS) in Figures 1-3 c gives the average of powers in each period. In analysis it can be used to test the significance of the oscillation variance of a particular period in the original dielectric value series, since wavelet transform conserves variance [1].

Examples

The following Figures 1-3 describe examples of the result of the wavelet analysis technique of Torrence and Compo when it is applied to two empirical and one theoretical data series. All the figures are made by an interactive Wavelet Plot server [2]. The following options were used for CWT:

Wavelet function	Paul, parameter 4
Wavelet scale	0.75

The Paul wavelet function is complex non-orthogonal function. The plot of the real part (solid) and imaginary part (dashed) can be seen in Figures 1-3. The imaginary part can be used in wavelet transform to return phase information of oscillations in time series. In WPS analysis the choice of wavelet function is not critical. The parameter of Paul wavelet changes the width of the function which has influence on time and frequency resolutions. The narrower wavelet function is, the better time resolution and the worse frequency resolution are and the other way round.

The wavelet scale determines how the dilation of wavelet function is performed in transform. The choice of scale has no special significance for WPS analysis, but the conversion of wavelet scale to the detected period in time series, i.e to the Fourier period, is convenient to do in the scalogram plot. [1, pp 66-68]

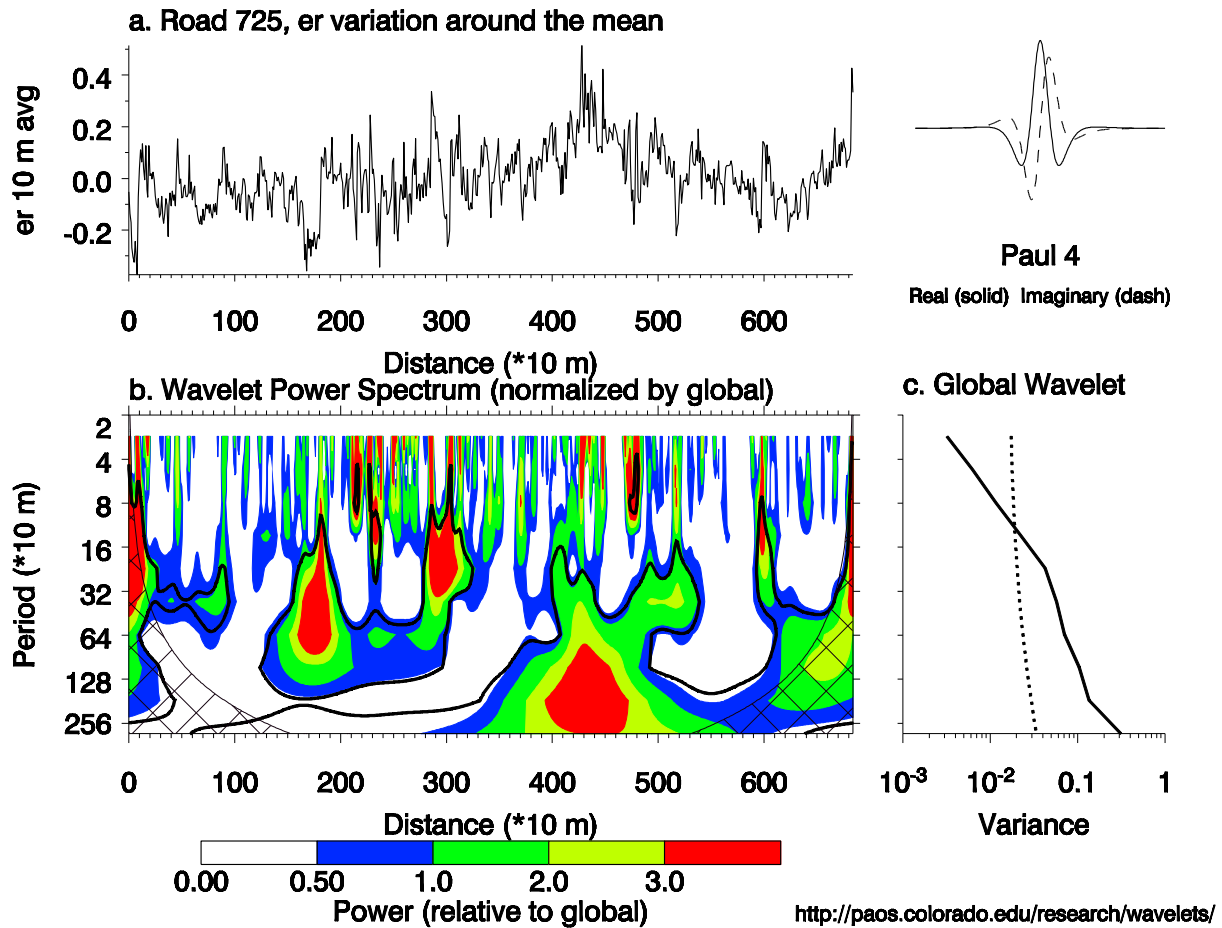


Figure 1 (Text is from the interactive server plot [2]): (a) Road 725, er variation around the mean . (b) The wavelet power spectrum. The power has been scaled by the global wavelet spectrum (at right). The cross-hatched region is the cone of influence, where zero padding has reduced the variance. Black contour is the 5% significance level, using a white-noise background spectrum. (c) The global wavelet power spectrum (black line). The dashed line is the significance for the global wavelet spectrum, assuming the same significance level and background spectrum as in (b). Reference: Torrence, C. and G. P. Compo, 1998: [A Practical Guide to Wavelet Analysis](#). *Bull. Amer. Meteor. Soc.*, **79**, 61-78.

The spectrum in Figure 1b reveals, for example, that the sections from 1500 m to 2000 m, from 2700m to 3100 includes significantly wide variation in dielectric value with large scale of periods from 80 m to 640 m and from 20m to 320 m respectively. The global wavelet power spectrum in Figure 1c shows that the variance in the large 160-2580 m band is significantly above the 95% confidence level (5 % significance level) for white noise.

The cone of influence that is marked by cross-hatched region illustrates the area where errors in power will occur because of the beginning and the end of finite-length time series.

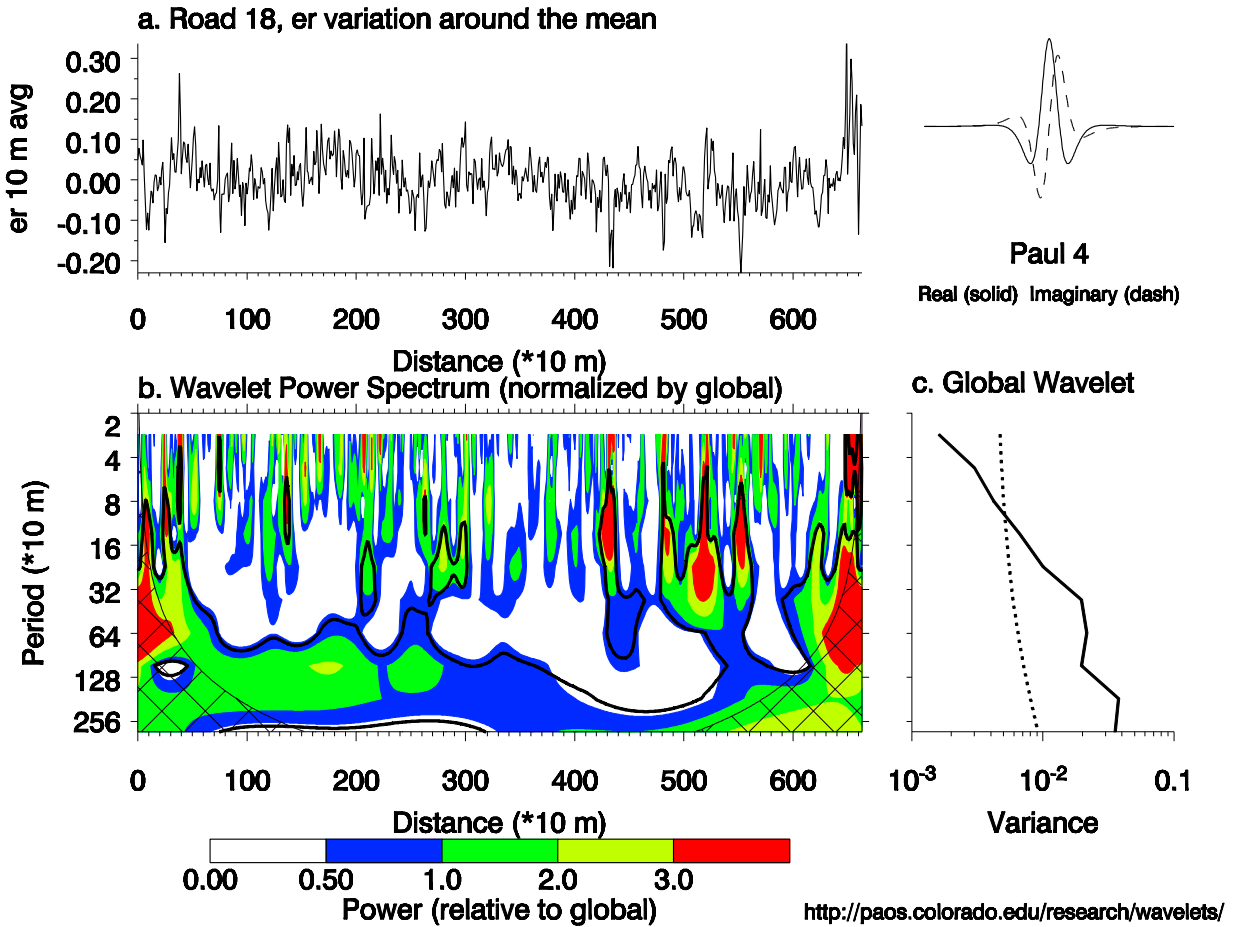


Figure 2 ϵ_r variation around the mean on road 18.

The spectrum in Figure 2b illustrates quite uniform ϵ_r distribution. The significantly wide variations of periods from 16 m to 100 m at the beginning and end of scalogram lie on the cone of influence and should be interpreted carefully. The global wavelet power spectrum in Figure 2c shows similar result for the variance relating to 95% confidence level for white noise. However, the 95% confidence level for white noise in Figure 2c is much below the one in Figure 1c. If we compare the global wavelet spectrum of road 18 to 95% confidence level of Figure 1c, we can see that variance lies almost entirely below that level. The

theoretical details for computing 95% confidence level for white noise can be seen in [1, pp 69-71].

In Figure 3 WPS analysis is applied to theoretical Gaussian white noise distribution with mean = 0 and standard deviation = 0.12535. The standard deviation is the same as the one of ε_r -distribution of road 725 in Figure 1a. Gaussian white noise is an independent white noise in which values are all normally distributed, and which could be regarded as a uniform ε_r distribution on a road section. The normalized power spectrum and the global wavelet spectrum illustrate therefore an example of good quality in new asphalt pavement based on homogeneity.

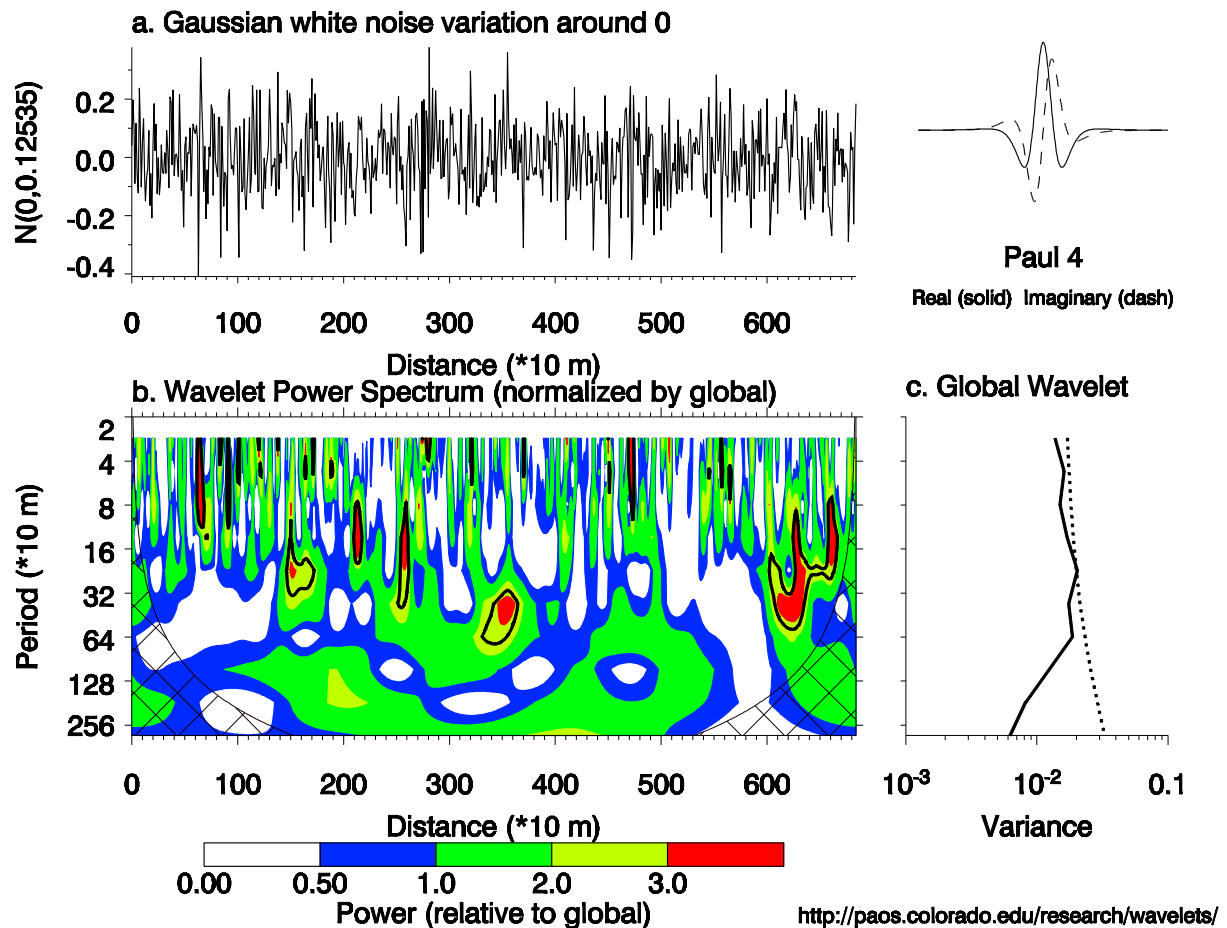


Figure 3: Gaussian white noise distribution $N(0, 0.12535)$.

Conclusions

After all, there is a need for further research to make quantitative interpretations for the uniformity of ϵ_r –distribution of ground penetrating radar (GPR) survey, i.e. for the homogeneity of new asphalt pavement, by the wavelet analysis technique introduced by Torrence and Combo [1, 2]. First of all, the adequate background noise spectrum should be investigated to determine the significance level that can be used as a limiting level of good homogeneity of asphalt pavement.

The Frequently Asked Questions in [2] have some instructions that might be useful for wavelet analysis of dielectric value variation. The proportion of the area above 5% significance level should be compared to that expected by chance. An adequate Gaussian white noise spectrum can be used here. From wavelet analysis we can find whether the 5% regions are distributed randomly or with some pattern. This may be used for revealing possible segregation. The reliability of wavelet analysis can be improved by Fourier analysis especially to detect the significant oscillation frequencies.

Wavelet spectrum analysis can be applied also to direct pulse amplitude variation. The spectrum reveals the long term changes in the amplitude which indicates changes in gaining and should be taken into account in dielectric value calculations. An interesting question is whether wavelet transform with inversion can be used to eliminate the influence of varying gaining from the dielectric value variation. Inverse wavelet transform can be used to reconstruct the original time series into a form where some desired features have been removed or changed.

References

- [1] Torrence, C. & Compo, G. P. (1998) A Practical Guide to Wavelet Analysis. Bull. Amer. Meteor. Soc., Vol. 79, No. 1, January 1998: 61-78.
- [2] Torrence, C. & Compo, G. P. A Practical Guide to Wavelet Analysis. Referred 20.2.2012.
<http://ion.researchsystems.com/IONScript/wavelet/>
- [3] Addison, P.S. 2002. The Illustrated Wavelet Transform Handbook. Taylor & Francis Group, LLC.